International Journal of Management, IT & Engineering
Vol. 8 Issue 8, August 2018,
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

FUZZY TIME SERIES FORECASTING MODEL USING COMBINED MODELS

V.Vamitha*

Abstract

Since nineteenth century, many researchers proposed various methods for forecasting enrollments, temperature prediction, stock price etc. In this paper a new model is introduced to forecast the fuzzy time series. By using Markov chain model the adjusted forecasted values are obtained and with aggregated fuzzy relationship the forecasted values are obtained. Using these two values,the proposed method is used to improve the accuracy. By giving example, forecasting the fuzzy time series are explained. The University of Alabama is used for illustration. Finally error analysis is made and error percentage is calculated and compared with existing methods. The proposed model confirms the potential benifits of the proposed approach with very small AFER.

Keywords:

Fuzzy time series model;
Markov chain;
fuzzy logic group;
latest information;
over all information.

Author correspondence:

V.Vamitha
Department of mathematics, B.C.M.W.G.Polytechnic college, Ettayapuram
Email: vvamitha80@gmail.com

1. Introduction

Markov chains are useful tools in modeling many practical systems such as queuing systems, manufacturing systems and inventory systems. Song and Chissom [19] proposed fuzzy time series models and fuzzy forecasting to model and forecast process whose observations are linguistic values. They illustrate the methodology by forecasting the enrollment at the University of Alabama from 20 years of data. Song and Chissom [19] used time-invariant fuzzy time series model, while Song and Chissom [20] used a time-variant model for the same problem. In their example the crisp data were fuzzified into linguistic values to illustrate the fuzzy time series method using fuzzy set theory. They asserted that all traditional forecasting methods fail when the enrollment data are composed of linguistic values. However a Markov model, described by Sullivan and Woodall [21] can use linguistic labels directly, but with the membership functions of the fuzzy approach replaced by analogous probability functions. Traditionally, the parameters of Markov model are estimated from observations in which the state occupied is known with certainty. Both the fuzzy forecasting and Markov models use the linguistic values directly to produce a 'fuzzy' forecast that, in turn, is 'defuzzified ' into a numerical point estimate. A discrete-state Markov process has a (possibly infinite) number of states or categories that are mutually exclusive. The Markov property requires that the probability of transition to a particular state j, given the process is currently in state i, be independent of the history of states occupied before the current state. Typically with a finite number of states these transition probabilities are arrayed in a p × p transition matrix, where p is the number of states in the model. The element in row i and column j gives the probability of transition to state j given the current state is i. If we let P_i denote the

* Doctorate Program, Linguistics Program Studies, Udayana University Denpasar, Bali-Indonesia

vector of state probabilities at time t, where entry n in this vector gives the probability that the system is in state n at t, then

$P'_{t+1} = P'_t * R_m$,

where R_m is the transition matrix. The product $P'_t * R^k_m$ gives the vector of state probabilities for time t + k. Thus, in general,

$P'_{t+k} = P'_t * R^k_m$, k = 1, 2,…

The transition matrix R_m may vary with time, in which case a subscript is added to indicate the time to which it applies. Even though the state of a Markov chain is mutually exclusive, the process being modeled does not have to occupy one particular state with certainty for a Markov model to be valid. Several states can have non-zero probabilities, analogous to the concept of fuzzy set membership. With the Markov model, however, there is the requirement that all the state probabilities must sum to one for each observation. Membership functions do not have this restriction. From the experimental result of fuzzy time series available in the literature, the forecasting accuracy is based on effective length of intervals, fuzzy logical relationships and datum considered. We observe the forecasting accuracy is impacted not only by the fuzzy logical relationships but also by the latest historical data. Therefore, it may reduce the forecasting accuracy when the variation of latest time cannot be accounted into forecasting factors. To reconcile this problem, first a new method which aggregates overall information of fuzzy logical relationships with latest information should be developed to find forecasting values.

The experimental results show that the proposed model has proved an effective tool in the prediction of enrolment. The paper is organised as follows. Section 2 introduces the concept of the fuzzy time series model. Section 3 proposes a fuzzy time series-Markov chain model and aggregated fuzzy relationship. Section 4 presents an illustration forecasting the enrolment at the university of Alabama and comparison of error with the existing results. Section 5 summarises the conclusion.

2. Basic concept of fuzzy time series.

Definition 1: Let Y(t) (t = 0,1,2,3,…) be a subset of R, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1,2,3…) are defined and let F(t) be the collection of $f_i(t)$. Then, F(t) is called as fuzzy time series on Y(t)(t = 0,1,2,3,…). From this definition, we can see that

(1) F(t) is a function of time

(2) F(t) can be regarded as a linguistic variable, which is a variable whose values are linguistic values represented by fuzzy sets.

(3) $f_i(t)$ (i = 1,2,3, …) are possible linguistic values of F(t), where f_i (t) (i = 1,2,3,….) are represented by fuzzy sets.

Definition 2: Suppose F(t) is caused only by F(t – 1) and is denoted by F(t –1) → F(t) , then there is a fuzzy relationship between F(t) and F(t –1) and can be expressed as the fuzzy relational equation F(t) = F(t –1) ∘ R(t, t –1). Here '∘' is max- min composition operator. The relation R is called *first – order model of F(t).*Further, if fuzzy relation R(t, t –1) of F(t) is independent of time t, that is to say, for different times t_1 and t_2,

$$R(t_1, t_1-1) = R(t_2, t_2-1),$$

then F(t) is called a *time invariant fuzzy time series,* otherwise F(t) is *time variant***.**

Definition 3: Suppose F(t –1) = A_i and F(t) = A_j. A fuzzy logical relationship can be defined as $A_i \rightarrow A_j$ where A_i and A_j are called the left hand side and the right hand side of the fuzzy logical relationship respectively.

If there are fuzzy logical relationships is obtained from A_i as $A_i \rightarrow A_j$, $A_i \rightarrow A_k$, $A_i \rightarrow A_l$, the fuzzy logical relationships are grouped into a fuzzy logical relationship group [19] as $A_2 \rightarrow A_j, A_k, A_l$.

3.Proposed method:

Step 1: Define the universe of discourse U, based on the range of available historical time series data by the following rules:

$U = [D_{min} - D_1, D_{max} + D_2]$, where D_1 and D_2 are two proper positive numbers.

Step 2: Partition the universe of discourse in to equal length of intervals: $u_1, u_2, \dots, u_m$ .The number of intervals will be in accordance with the number of linguistic values (fuzzy sets) $A_1, A_2, \dots, A_m$ to be considered.

Step 3: Define fuzzy sets on the universe of discourse U. There is no restriction on determining how many linguistic variables can be fuzzy sets. Thus, the "enrollment" can be described by the fuzzy sets of A_1 = (not many), A_2 = (not too many), A_3 = (many), A_4 = (many many), A_5 = (very many), A_6 = (too many), A_7 = (too many many). For simplicity, each fuzzy set A_i (i = 1, 2, ..., 7) is defined on seven intervals, which are $u_1 = [d_1, d_2]$, $u_2 = [d_2, d_3]$, $u_3 = [d_3, d_4]$, $u_4 = [d_4, d_5]$, ... , $u_7 = [d_7, d_8]$; thus, the fuzzy sets $A_1, A_2, \dots, A_7$ are defined as follows:

$$A_1 = \{1/u_1, 0.5/u_2, 0/u_3, 0/u_4, 0/u_5, 0/u_6, 0/u_7\}$$
$$A_2 = \{0.5/u_1, 1/u_2, 0.5/u_3, 0/u_4, 0/u_5, 0/u_6, 0/u_7\}$$
$$A_3 = \{0/u_1, 0.5/u_2, 1/u_3, 0.5/u_4, 0/u_5, 0/u_6, 0/u_7\}$$
$$A_4 = \{0/u_1, 0/u_2, 0.5/u_3, 1/u_4, 0.5/u_5, 0/u_6, 0/u_7\}$$
$$A_5 = \{0/u_1, 0/u_2, 0/u_3, 0.5/u_4, 1/u_5, 0.5/u_6, 0/u_7\}$$
$$A_6 = \{0/u_1, 0/u_2, 0/u_3, 0/u_4, 0.5/u_5, 1/u_6, 0.5/u_7\}$$
$$A_7 = \{0/u_1, 0/u_2, 0/u_3, 0/u_4, 0/u_5, 0.5/u_6, 1/u_7\}$$

Step 4: Fuzzify the historical data and establish the π-order fuzzy relationship $A_{t-\pi}, A_{t-\pi+1}, \dots, A_{t-2}, A_{t-1} \rightarrow A_t$, where $\pi \geq 1$, $t \geq 2$ and where A_{t-1}, A_t denote the latest past in the current state and the next state respectively. This step aims to find an equivalent fuzzy set for each input data. If the collected time series data belongs to an interval u_i, i = 1, 2,…,7, then it is fuzzified to the fuzzy sets A_i.

Step 5: Determine π-order, $\pi \geq 1$ fuzzy logical relationship groups. All fuzzy relationships with the same current state are put together to form fuzzy relationship groups.

Step 6: We consider two cases to forecast the output values namely Markov state transition matrix method and aggregate method.

Case (i) : **(By Markov state transition matrix method)**

(a)Calculating forecasting values:

We define n states for each time step for n fuzzy sets to establish $n \times n$ Markov state transition matrix. If state A_i makes a transition in to state A_j and passes another state A_k, i, j = 1, 2, …, n , then we can obtain the fuzzy logical relationship group. The transition probability of state [17] is written as

$p_{ij} = \frac{T_{ij}}{T_i}$, i, j = 1, 2, … , n. where p_{ij} is the probability of transition from state A_i to A_j by one step, T_{ij} is the transition times from state A_i to A_j by one step, and T_i is the amount of data belonging to the A_i state. Then the transition probability matrix R of the state can be written as

$$R = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}$$

Definition 3.2.1 [17]: If $p_{ij} \geq 0$, then state A_j is reached from state A_i. If the state A_i is reached from state A_j, then we say that the states i and j are *communicate*.(i.e.) A_i communicates with A_j. If a transition occurs from A_i to A_j, with probability $p_{ij} \geq 0$, j = 1, 2,…,n , then $\sum_j p_{ij} = 1$.

If F(t−1) = A_i, the process is defined to be in state A_i at time t − 1; then forecasting of F(t) is conducted using the row vector [$P_{i1}, P_{i2}, \dots, P_{in}$]. The forecasting of F(t) is equal to the weighted average of $m_1, m_2, \dots, m_n$, the midpoint of $u_1, u_2, \dots, u_n$. The expected forecasting values are obtained by the following Rules:

Rule 1 : If the fuzzy logical relationship group of A_i is one-to-one (i.e.,$A_i \rightarrow A_k$, with $P_{ik} = 1$ and $P_{ij} = 0, j \neq k$), then the forecasting of F(t) is m_k, the midpoint of u_k, according to the equation $F(t) = m_k P_{ik} = m_k$.

Rule 2: If the fuzzy logical relationship group of A_j is one-to-many (i.e., $A_j \rightarrow A_1, A_2, \dots, A_n$, j = 1,2, . . . ,n), when collected data Y(t−1) at time t−1 is in the state A_j, then the forecasting of F(t) is equal as $F(t) = m_1P_{j1} + m_2P_{j2} + \dots + m_{j-1}P_{j(j-1)} + Y(t-1)P_{jj} + m_{j+1}P_{j(j+1)} + \dots + m_nP_{jn}$ where $m_1, m_2, \dots, m_{j-1}, m_{j+1}, \dots, m_n$ are the midpoints of $u_1, u_2, \dots, u_{j-1}, u_{j+1}, \dots, u_n$ and Y(t−1) is substituted for m_j in order to take more information from th state A_j at time t−1.

(b) Adjusted forecasting values:

Adjust the tendency of the forecasting values. For any time series experiment, a large sample size is always necessary. Therefore, under a smaller sample size when modeling a fuzzy time series-Markov chain model, the derived Markov chain matrix is usually biased, and some adjustments for the forecasting values are suggested to revise the forecasting error. First, in a fuzzy logical relationship group where A_i communicates with A_i and is one-to-many, if a large state A_j is accessible from state A_i, i, j = 1,2,…,n, then the forecasting value for A_j is usually underestimated because the lower state values are used for forecasting the value of A_j. On the other hand, an overestimated value should be adjusted for forecasting value A_j because a smaller state A_j is accessible from A_i, i, j = 1, 2, . . . , n. Second, any transition that jumps more than two steps from one state to another state will derive a change-point forecasting value, so that it is necessary to make an adjustment to the forecasting value in order to obtain a smoother value. That is, if the data happens in the state A_i, and then jumps forward to state A_{i+k} ($k \geq 2$) or jumps backward to state A_{i-k} ($k \geq$

2), then it is necessary to adjust the trend of the pre-obtained forecasting value in order to reduce the estimated error. The adjusting rule for the forecasting value is described below.

Rule1 **:** If state A_i communicates with A_i, starting in state A_i at time $t-1$ as $F(t-1) = A_i$ and makes a transition into state A_{i+1} at time t, then the adjusted forecasting value is $F'(t) = F(t) + (l/2)$,where l is the length of the interval.

Rule 2**:** If state A_i communicates with A_i starting in state A_i at time $t-1$ as $F(t-1) = A_i$ and makes a transition into state A_{i-1} at time t, then the adjusted forecasting value is $F'(t) = F(t) - (l/2)$.

Rule 3**:** If the current state is in state A_i and makes a forward transition into state A_j at time t and A_i communicates with A_j then the adjusted forecasting value is $F'(t) = F(t) + 2(l/2)$.

Rule 4**:** If the current state is in state A_i at time $t-1$ as $F(t-1) = A_i$ and makes a backward transition into state A_j at time t and A_i communicates with A_j, then the adjusted forecasting value is

$$F'(t) = F(t) - 2(l/2).$$

Rule 5**:** If the current state is in state A_i at time $t-1$ as $F(t-1) = A_i$ and makes a jump forward transition into state A_{i+s} at time t $(1 < s \leq n-i)$ then the adjusted forecasting value is

$$F'(t) = F(t) + (l/2)\, s.$$

Rule 6**:** If the current state is in state A_i at time $t-1$ as $F(t-1) = A_i$ and makes a jump backward transition into state A_{i-s} at time t $(1 < s \leq i)$ then the adjusted forecasting value is

$$F'(t) = F(t) - (l/2)\, s.$$

Case (ii) : (By Aggregate method) Calculating forecasting values:

To improve the forecasting accuracy, we discuss a new forecasting method which aggregates the overall information and latest variation on fuzzy relationships to calculate the forecasted value. As shown in equation (1), w_1 and w_2 are adaptive weights for overall information of the fuzzy relationships and latest information respectively. For each group of the fuzzy rule, the forecasted value consists of two weighted parts on overall information and latest information respectively. Here by, the forecasted value of enrollments is represented as:

Forecasted value = $w_1 \times$ *overall information* + $w_2 \times$ *latest information*, (1)

where $w_1 + w_2 = 1$ and $0 \leq w_1, w_2 \leq 1$. Without loss of generality, we assume w_1 and w_2 are equally weighted. Initially, we consider training data which has known current state and next state. In equation (1), the overall information can be decided by the fuzzy groups created in step 5.

Based on the adopted method of defuzzification by Chen (1996), the defuzzified value can be computed by the midpoint of the next state for each fuzzy relationship. By applying seven intervals in step 2, the midpoint m_t of each interval u_t can be calculated as follows: $m_t = (bs_t - be_t)/2$, where $1 \leq t \leq 7$ and u_t is bounded within $(bs_t, be_t]$. If a fuzzy relationship group consists of more than one fuzzy relationship, the value of overall information is the average of the corresponding midpoints of all intervals with respect to all linguistic values existing in the next states of all fuzzy relationships. Assume a first-order fuzzy relationship group is $A_{t-1} \rightarrow A_{t1}, A_{t2}, \ldots, A_{tk}$, and $m_{t1}, m_{t2}, \ldots, m_{tk}$ are the midpoints of linguistic values $A_{t1}, A_{t2}, \ldots, A_{tk}$, respectively. Then, the value of overall information is calculated as follows:

$$overall\ information = \frac{m_{t1} + m_{t2} + \ldots + m_{tk}}{k} \qquad (2)$$

The latest information is derived by the latest fuzzy variation scheme. This scheme is an estimating scheme determined by the next state and the latest past in the current state. Assume a π-order fuzzy relationship is $A_{t-\pi}, A_{t-\pi+1}, \ldots, A_{t-2}, A_{t-1} \rightarrow A_t$, where $\pi \geq 1$ and $t \geq 2$, where A_{t-1} and A_t denote the latest past in the current state and the next state, respectively. Here, m_{t-1} and m_t are midpoints of the fuzzy intervals u_{t-1} and u_t with respective to A_{t-1} and A_t where $u_{t-1} = (bs_{t-1}, be_{t-1}]$. $u_t = (bs_t, be_t]$. As shown in equation (3), the latest variation scheme calculates fuzzy difference between A_{t-1} and A_t using m_{t-1} and m_t; then the fuzzy difference should be normalized by dividing $m_{t-1} + m_t$. The complete latest variation scheme is formulated as follows:

$$Latest\ information = bs_t + \frac{be_t - bs_t}{2} \times \frac{m_t - m_{t-1}}{m_t + m_{t-1}}. \qquad (3)$$

Here we take the model of the first-order as framing rules on fuzzy relationships.

Step 7: Choose F_i by the rule (i) If $|A_i - F'_i| < |A_i - f_i|$ choose $F_v = F'_i$ (or) rule (ii) If $|A_i - F'_i| > |A_i - f_i|$ choose $F_v = f_i$.

Step 8: Choose $\propto$ in (0, 1). Make an error analysis as follows:

Final forecasted value $= \frac{Actual\ value + New\ forecasted\ value - \propto}{2}$

The AFER is used to measure the accuracy as a percentage as follows:

$$AFER = \frac{1}{n}\sum_{i=1}^{n}\frac{|A_i - F_i|}{A_i} * 100.$$

4. Results and Analysis

Step 1 : Define universe of discourse U and partition it into seven equal-length intervals, The collected data is shown in the second column of table 1; we have the enrollments of the university from 1971 to 1992 with D_{min} = 13055 and D_{max} = 19337. We choose D_1 = 55 and D_2 = 663. Thus, U = [13000, 20000].

Step 2 : U is partitioned into seven intervals with u_1 = [13000, 14000], u_2 = [14000, 15000], u_3 = [15000, 16000], u_4 = [16000, 17000], u_5 = [17000, 18000], u_6 = [18000, 19000], u_7 = [19000, 20000]. The mid-values of the intervals are m_1 = 13500, m_2 = 14500, m_3 = 15500, m_4 = 16500, m_5 =17500, m_6 = 18500, m_7 = 19500.

Step 3: The fuzzy sets corresponding to the enrollments of the university from 1971 to 1992 are given in table 1.

Table 1: The enrollment data and corresponding fuzzy sets

Year	Actual Enrollment	Fuzzy sets	Year	Actual Enrollment	Fuzzy sets
1971	13055	A_1	1982	15433	A_3
1972	13563	A_1	1983	15497	A_3
1973	13867	A_1	1984	15145	A_3
1974	14696	A_2	1985	15163	A_3
1975	15460	A_3	1986	15984	A_3
1976	15311	A_3	1987	16859	A_4
1977	15603	A_3	1988	18150	A_6
1978	15861	A_3	1989	18970	A_6
1979	16807	A_4	1990	19328	A_7
1980	16919	A_4	1991	19337	A_7
1981	16388	A_4	1992	18876	A_6

Step 4:

Table 2: The first-order and three-order fuzzy relationship on enrollment

Years	Fuzzy sets	First-order	Three-order
1971	A_1		
1972	A_1	$A_1 \rightarrow A_1$	
1973	A_1	$A_1 \rightarrow A_1$	
1974	A_2	$A_1 \rightarrow A_2$	$A_1, A_1, A_1 \rightarrow A_2$
1975	A_3	$A_2 \rightarrow A_3$	$A_1, A_1, A_2 \rightarrow A_3$
1976	A_3	$A_3 \rightarrow A_3$	$A_1, A_2, A_3 \rightarrow A_3$
1977	A_3	$A_3 \rightarrow A_3$	$A_2, A_3, A_3 \rightarrow A_3$
1978	A_3	$A_3 \rightarrow A_3$	$A_3, A_3, A_3 \rightarrow A_3$

1979	A_4	$A_3 \rightarrow A_4$	$A_3, A_3, A_3 \rightarrow A_4$
1980	A_4	$A_4 \rightarrow A_4$	$A_3, A_3, A_4 \rightarrow A_4$
1981	A_4	$A_4 \rightarrow A_4$	$A_3, A_4, A_4 \rightarrow A_4$
1982	A_3	$A_4 \rightarrow A_3$	$A_4, A_4, A_4 \rightarrow A_3$
1983	A_3	$A_3 \rightarrow A_3$	$A_4, A_4, A_3 \rightarrow A_3$
1984	A_3	$A_3 \rightarrow A_3$	$A_4, A_3, A_3 \rightarrow A_3$
1985	A_3	$A_3 \rightarrow A_3$	$A_3, A_3, A_3 \rightarrow A_3$
1986	A_3	$A_3 \rightarrow A_3$	$A_3, A_3, A_3 \rightarrow A_3$
1987	A_4	$A_3 \rightarrow A_4$	$A_3, A_3, A_3 \rightarrow A_4$
1988	A_6	$A_4 \rightarrow A_6$	$A_3, A_3, A_4 \rightarrow A_6$
1989	A_6	$A_6 \rightarrow A_6$	$A_3, A_4, A_6 \rightarrow A_6$
1990	A_7	$A_6 \rightarrow A_7$	$A_4, A_6, A_6 \rightarrow A_7$
1991	A_7	$A_7 \rightarrow A_7$	$A_6, A_6, A_7 \rightarrow A_7$
1992	A_6	$A_7 \rightarrow A_6$	$A_6, A_7, A_7 \rightarrow A_6$

Step 5:

Table 3: The first-order fuzzy relationship groups on enrollments

Groups	First-order fuzzy relationship groups
G_1	$A_1 \rightarrow A_1, A_2$
G_2	$A_2 \rightarrow A_3$
G_3	$A_3 \rightarrow A_3, A_4$
G_4	$A_4 \rightarrow A_4, A_3, A_6$
G_5	$A_6 \rightarrow A_6, A_7$
G_6	$A_7 \rightarrow A_7, A_6$

Table 4: The three-order fuzzy relationship groups on enrollments

Groups	Three-order fuzzy relationship groups
G_1	$A_1, A_1, A_1 \rightarrow A_2$
G_2	$A_1, A_1, A_2 \rightarrow A_3$
G_3	$A_1, A_2, A_3 \rightarrow A_3$
G_4	$A_2, A_3, A_3 \rightarrow A_3$
G_5	$A_3, A_3, A_3 \rightarrow A_3, A_4$
G_6	$A_3, A_3, A_4 \rightarrow A_4, A_6$
G_7	$A_3, A_4, A_4 \rightarrow A_4$
G_8	$A_4, A_4, A_4 \rightarrow A_3$
G_9	$A_4, A_4, A_3 \rightarrow A_3$
G_{10}	$A_4, A_3, A_3 \rightarrow A_3$
G_{11}	$A_3, A_4, A_6 \rightarrow A_6$
G_{12}	$A_4, A_6, A_6 \rightarrow A_7$
G_{13}	$A_6, A_6, A_7 \rightarrow A_7$

G_{14}	$A_6, A_7, A_7 \rightarrow A_6$

Step 6:

Case(i):

Using the fuzzy logical relationship group in table 2, the transition probability matrix R may be obtained.

$$R = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7/9 & 2/9 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 1/2 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

Adjust the tendency of the forecasting values. The relationships between the states are analyzed in Figure 1. It is clear that states 3 and 4 communicate with each other, thus an adjust value should be considered, or vice versa. By contrast, state 6 and state 7 also communicates with each other, but in the end these states uncertainty in relation to the future trend is larger and unknown; thus, we do not adjust the value.

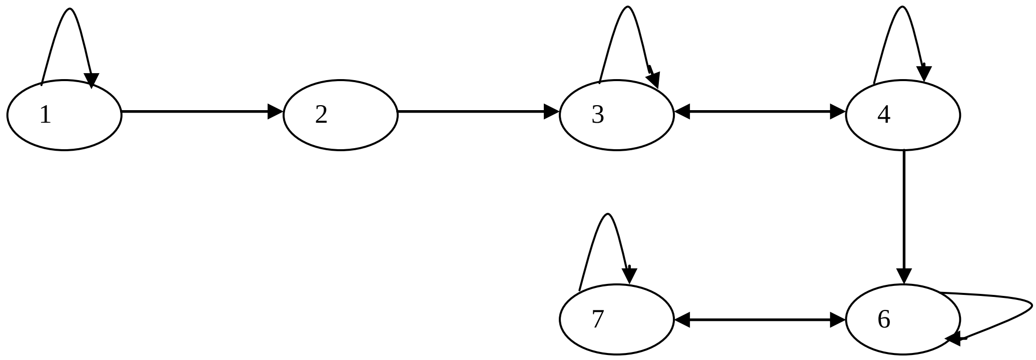

Figure 1: Transition process for enrollment forecasting

For case (i) :

Suppose we want to adjust the forecasted value for the year 1979. According to the Rule 2 of case (i) (a), the forecasting value of the year 1979.

$$\begin{aligned} F(1979) &= (7/9)\, Y(1978) + (2/9)\, m_4 \\ &= (7/9)(15861) + (2/9)\, 16500 \\ &= 16003. \end{aligned}$$

The corresponding fuzzy logical relationship is $A_3 \rightarrow A_4$. According to figure 1, the states 3 and 4 communicate with each other. Then by Rule 3 of case (i) (b), the adjusted forecasting value is

$$\begin{aligned} F'(1979) &= F(1979) + 2\left(l/2\right), \quad \text{where } l = 1000 \\ &= 16003 + 1000 \\ &= 17003. \end{aligned}$$

For case (ii):

Suppose we want to forecast the enrollment of year 1975. Based on table 1, we see that the linguistic enrollment of current state at 1974 is A_2. In table 3, we find that a fuzzy relationship $A_2 \rightarrow A_3$ in group G_2 appears the same linguistic value of the current state A_2. The maximum membership values of the fuzzy sets A_2 and A_3 occur at intervals u_2 and u_3, respectively, where $u_2 = (bs_2, be_2]$ and $u_3 = (bs_3, be_3]$. From step 2, we obtained $bs_2 = 14000$, $be_2 = 15000$, $bs_3 = 15000$ and $be_3 = 16000$. The midpoints of the intervals u_2 and u_3 are $m_2 = 14500$ and $m_3 = 15500$, respectively, where $m_2 = ½(14000 + 15000)$ and $m_3 = ½(15000 + 16000)$. The overall information of year 1975 is equal to m_3, that is overall information =15500. According to equation (3), by setting $bs_t = bs_3$, $be_t = be_3$, $m_{t-1} = m_2$, $m_t = m_3$, $u_{t-1} = u_2$ and $u_t = u_3$, we can calculate the value of the latest information on the enrollment of year 1975 as follows:

$$\begin{aligned} Latest\ information &= bs_3 + \frac{be_3 - bs_3}{2} \times \frac{m_3 - m_2}{m_3 + m_2} \\ &= 15000 + \frac{16000 - 15000}{2} \times \frac{15500 - 14500}{15500 + 14500} \end{aligned}$$

= 15017.

Moreover, assume we want to forecast the enrollment of year 1982, we can see that the current state of the enrollment of year 1981 is A_4 in table 2, we can find that three-order fuzzy relationships $A_4 \to A_4$, $A_4 \to A_3$, and $A_4 \to A_6$ in group G_4 appear the same current state A_4. Based on equations (1) to (3), the forecasted enrollment of the year 1982 can be calculated as follows:

Forecasted value = $w_1 \times$ *overall information* + $w_2 \times$ *latest information*

$$= w_1 \times \frac{m_4 + m_3 + m_6}{3} + w_2 \times \left(bs_3 + \frac{be_3 - bs_3}{2} \times \frac{m_3 - m_4}{m_3 + m_4}\right)$$

$$= 0.5 \times \frac{16500 + 15500 + 18500}{3} + 0.5 \times \left(15000 + \frac{16000 - 15000}{2} \times \frac{15500 - 16500}{15500 + 16500}\right)$$

= 15908.

The forecasted enrollments of the first-order fuzzy relationships are listed in table 5. In the case (i), adjust the forecasted value using the rules concerned in step 6 of case (i) (b). Tabulate the forecasting values which are nearer to the actual values from the two cases.

Table 5: Enrollment forecasting

Year	Historical data A_i	By case(i)		By case(ii)	New forecasted value F_v
		Forecasted value F_i	Adjusted forecasted value F'_i	Forecasted value f_i	
1971	13055	-	-	-	-
1972	13563	13537	13537	13500	13537
1973	13867	13875	13875	13500	13875
1974	14696	14078	14578	14009	14578
1975	15460	15500	15500	15258	15500
1976	15311	15691	15691	15500	15500
1977	15603	15575	15575	15500	15575
1978	15861	15802	15802	15500	15802
1979	16807	16003	17003	16008	17003
1980	16919	16904	16904	16416	16904
1981	16388	16960	16960	16416	16416
1982	15433	16694	15694	15908	15694
1983	15497	15670	15670	15500	15500
1984	15145	15720	15720	15500	15500
1985	15163	15446	15446	15500	15446
1986	15984	15460	15460	15500	15500
1987	16859	16099	17099	16008	17099
1988	18150	16930	17930	17431	17930
1989	18970	18600	18600	18500	18600
1990	19328	19147	19647	19006	19647
1991	19337	18914	18914	19000	19000
1992	18876	18919	18919	18493	18919

Step 7: Using AFER, we calculated the value of the proposed method. This calculated value and some other values available in the literature related to this work are tabulated in table 6.

Table 6: Comparison of forecasting errors for six types of methods

Method	Song and Chissom method (1993)	Tsaur et al. (2005)	Cheng et al. (2006)	Singh (2007)	Li and Cheng (2007)	Proposed model
AFER (%)	3.22	1.86	1.7236	1.5587	1.53	0.5065

4. Conclusion

In this paper, we have presented a model in academic enrollments based on Markov chain with aggregated fuzzy relationships. First, we find out the relationship matrix using Markov analysis and adjust the forecasted values. Secondly, we studied the effect of forecasting accuracy relates to the overall information and the latest variation. From the empirical study of forecasting enrollments of students of the University of Alabama, the experimental results show that the proposed model gets higher forecasting accuracy than any existing models tabulated in table 6. Further study can be made on the nature of states of the Markov matrix. Based on the behaviour of the states the fuzzy relationships may be aggregated.

References

[1].Box. G. E. P and Jenkins. J: "Time series analysis, Forecasting and Control", (1976) Holden- Day.

[2].Cerf. V.,Fernandez. E., Gostelow.K and Volausky. S: "Formal control and low properties of a model of computation", Report ENG 7178, Computer Science Department ,University of California, Los Angeles, CA, (1971),p 81.

[3]. Chang. P. T: "Fuzzy seasonality forecasting, Fuzzy Sets and Systems", 90, (1997), pp l-10.

[4].Chen.S. M: " Forecasting enrollments based on fuzzy time series", Fuzzy Sets and Systems, 81(3), (1996) ,pp 311–319.

[5].Chen. S. M: " Forecasting enrollments based on high-order fuzzy time Series", Cybernetics and Systems, An International Journal, 33(1), (2002) , pp 1–16.

[6].Chen. S.M and Hsu. C. C: "A new method to forecast enrollments using fuzzy time series", International Journal of Applied Science and Engineering, 2(3), (2004), pp 234-244.

[7].Chen.S. M., Wang. N. Y and Pan. J. S: "Forecasting enrollments using automatic clustering techniques and fuzzy logical relationships", Expert systems with Applications, 36, (2009), pp 11070-11076.

[8].Chen. S. M: " Forecasting enrollments based on fuzzy time series", Fuzzy Sets and Systems, 81(3), (1996), pp 311–319.

[9]Chen. S. M: " Forecasting enrollments based on high-order fuzzy time Series", Cybernetics and Systems, An International Journal, 33(1), (2002), pp 1–16.

[10]Huang. Y. L., Horng. S. J., He. M., Fan. P., Kao. T.W., Khan. M. K., Lai J. L and Kuo. I. H: "A hybrid forecasting model for enrollments based on aggregated fuzzy time series and particle swarm optimization", Expert Systems with applications, 38, (2011), pp 8014- 8023.

[11]. Huarng. K: "Effective lengths of intervals to improve forecasting in fuzzy time- series", Fuzzy Sets and Systems, 123(3) , (2001a) , pp 387-394.

[12].Huarng. K: Heuristic models of fuzzy time series for forecasting, Fuzzy Sets and Systems, 123(3), (2001b), pp 369-386.

[13] Huarng. K and Yu. H. K: "The application of neural networks to forecast fuzzy time series", Physica A, 363, (2006), pp 481-491.

[14]Huarng. K and Yu. T. H. K: Ratio-based lengths of intervals to improve fuzzy time series forecasting, IEEE Transactions on Systems, man, and Cybernetics-part b, Cybernetics, 36(2), (2006), pp 328-340.

[15]Rajaram. S and Sakthisree. P: "Forecasting enrollments using interval length, weightage factor and fuzzy logical relationships", International journal of statistics and systems, 5(2) ,(2010), pp 173-182.

[16].Ross. S. M:" Introduction to probability models", Academic Press, Newyork, USA. (2003)

[17]Song.Q and Chissom B.S : "Fuzzy time series and its models, Fuzzy Sets and Systems, 54(3), (1993 b), pp 269-277.

[18]Song. Q and Chissom. B.S: Forecasting enrollments with fuzzy time series - part I, Fuzzy Sets and Systems, 54(1), (1993 a) ,pp 1-9.

[19]Song. Q and Chissom B. S: Forecasting enrollments with fuzzy time series - part II, Fuzzy Sets and Systems, 62(1), (1994), pp 1-8.

[20]Sullivan. J and Woodall. W. H: A comparison of fuzzy forecasting and Markov modeling, Fuzzy Sets and Systems, 64(3), (1994), pp 279-293.